Homework Assignment I

Physics 105.2, Instructor: Petr Hořava

This assingment is due Friday, Feb 7. It consists of five separate small problems, three of which are elementary operations with vectors and matrices, while the remaining two are based directly on two solved problems that appear in [Marion-Thornton]. In the case of these last two solved problems, I strongly encourage you to try and solve them from scratch, before you check out their explicit solution in [MT]. The purpose of the homework is for you to brush up a little bit on your elementary Newtonian mechanics skills that you acquired in your Freshman Mechanics course.

In addition to solving the problems, there is some required reading: Sections 1.1 through 1.12 and Sections 2.1 – 2.4 of [MT]. I leave it up to you how many of the Solved Examples in those subsections you want to study in detail, but you should make sure that you are comfortable with all the basic concepts presented there.

About grading: at least for the time being, we will award points based on the famous 0-1-2 pattern – 0 points if you don't attempt the problem, 1 point if you attempt it, 2 if you solve the problem correctly. Adjustments to this scheme may be necessary for later Homework Assignments as the semester progresses and the Assignments become more involved.

So, here are the first five problems; enjoy!

1. [Problem 1-11. of Marion-Thornton]
   Consider three vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{z}$ in the three-dimensional vector space $\mathbf{R}^3$. Consider the *triple scalar product* $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z}$, where $\times$ is the cross (or "vector") product and $\cdot$ denotes the inner (="dot") product of vectors as discussed in class. Show that this triple scalar product can be written as

$$(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z} = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

   where the straight vertical lines on the right hand side denote the determinant of the 3×3 matrix consisting of the components v_i, w_i and z_i of the three vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{z}$ in an orthonormal basis.
2. [continuation of Problem 1-11. of [MT]]
   In the same setup as in the previous problem, show also that the triple scalar product is unaffected by the change of the scalar and vector (=cross) product operations, of by a change in the order of $\mathbf{v}$, $\mathbf{w}$, $\mathbf{z}$, as long as they are in cyclic order; that is,

$$(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z} = \mathbf{v} \cdot (\mathbf{w} \times \mathbf{z}) = \mathbf{w} \cdot (\mathbf{z} \times \mathbf{v}), \quad \text{etc.}$$

   We may therefore use the notation $\mathbf{vwz}$ to denote the triple scalar product. Finally, give a geometric interpretation of $\mathbf{vwz}$ by computing the volume of the parallelepiped defined by the three vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{z}$.

3. [Problem 1-14. of [MT]]
   Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \quad C \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}.$$

   Find the following:

$$\text{(a) } |AB|, \quad \text{(b) } AC, \quad \text{(c) } ABC, \quad \text{(d) } AB - B^t A^t.$$

   (Here A^t refers to the transposition of the matrix A, etc.)
4. [Solved Example 2.5 in [MT]]
   Find the position and velocity of a particle undergoing vertical motion in a medium having a retarding force proportional to the velocity. More precisely, consider a particle falling downward with an initial velocity v_0 from a hight h in a constant gravitational field, and in the presence of a retarding force proportional to the particle's velocity. Write down the equation of motion and solve it.
5. [Solved Example 2.6 in [MT]]
   Study the motion of a projectile in two dimensions (i.e., vertical distance x and horizontal distance y under the infuence of constant gravitational force along y, without considering air resistance. The projectile starts at initial velocity $\mathbf{v}$ at $y = 0$. Calculate the projectile's position, velocity, and range (=x coordinate of the impact point where the projectile hits the ground at $y = 0$). As a voluntary bonus, you can add the influence of air resistance [see Example 2.7 of [MT]].